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A Proposed Lumped-Element Switching Circulator Principle

REINHARD H. KNERR, MEMBER, IEEE

Abstract—Two different analytical methods, the complex conjugate input admittance approach and the eigenvalue analysis, show the possibility of building a fast switching lumped-element circulator. In conventional switching circulators, switching is achieved by changing the required magnetic biasing field. The proposed principle, which is valid for circulators of all types, is especially interesting for lumped-element circulators where the switching may be accomplished by simply changing two capacitor values. The capacitors could be switched by varying voltages on semiconductors thus permitting very fast switching. The analysis has been experimentally verified. No attempt to obtain optimization of a specific design was made.

Introduction

IN THE COURSE of efforts to develop a high-performance photo-processed lumped-element circulator and appropriate analysis [1]–[3], it was discovered that it should be possible to switch the sense of circulation by switching parameters other than the magnetic biasing field. While this observation is valid in principle for circulators of all types, it is especially interesting for lumped-element circulators where the switching may be accomplished simply by changing lumped capacitors. In principle the capacitors could be switched by varying voltages on semiconductors, thus permitting very fast switching. The possibility of such a switching circulator was treated in passing in [2] and [3]. This paper will expand upon the analysis of the device.

In 1965 Konishi [4] and Dunn and Roberts [5] published papers describing lumped-element circulators at the heart of which were three inductors coupled through a common ferrite disk and resonated by individual capacitors. Various approaches have been taken to analyze this basic circulator type [1]–[8]. The author involves [1] an extension of Deutsch and Wieser's method [7] that will be referred to as the complex input-admittance method. The analysis of the more complex structures studied by the author is reported in [2] and [3]. This is an eigenvalue analysis that has been found extremely valuable in providing a fundamental understanding of the circulator operation and near quantitative performance predictions.

In this paper, each of these approaches will be used to demonstrate the principle of capacitive switching. References [2] and [3] will be relied upon for details of the eigenvalue analysis. Since [1] does not give any details of the complex input-admittance analysis, it will be outlined in this paper.

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The author is with Bell Laboratories, Inc., Allentown, Pa. 18103.

I. The Complex Input-Admittance Analysis

There is a well-known theorem [9] that states: a lossless three-port can only be matched at all three ports if it contains a lossless nonreciprocal element, and such a matched three-port represents an ideal circulator.

If the three-port in Fig. 1 is represented by

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \tag{1}$$

i.e., $[V] = [Z][i]$, then this three-port is lossless if

$$\text{Re}(\alpha) = 0 \quad \text{and} \quad \beta = -\gamma^* \tag{2}$$

where γ^* designates the complex conjugate of γ. It has been shown that the impedance matrix of the three-port

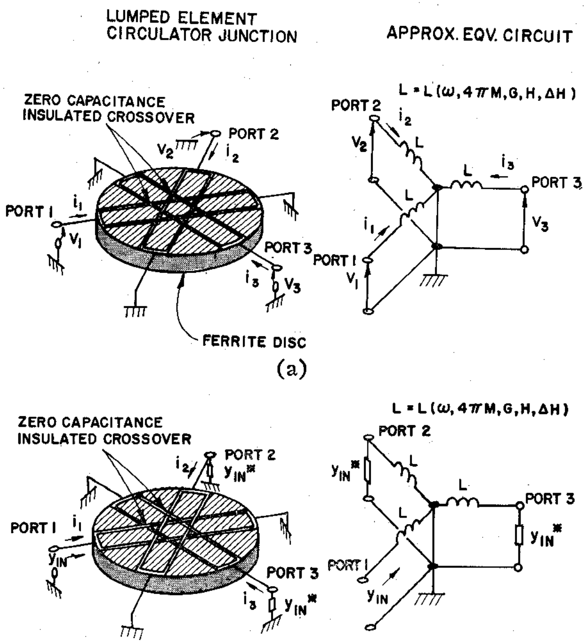


Fig. 1. (a) Lumped-element circulator junction and approximate equivalent circuit. (b) Lumped-element circulator junction and approximate equivalent circuit terminated by the complex conjugate of the input admittance.

in Fig. 1(a) is given by [7]

$$[Z] = \omega G \begin{bmatrix} j\mu & -j\frac{\mu}{2}-\frac{\kappa}{2}\sqrt{3} & -j\frac{\mu}{2}+\frac{\kappa}{2}\sqrt{3} \\ -j\frac{\mu}{2}+\frac{\kappa}{2}\sqrt{3} & j\mu & -j\frac{\mu}{2}-\frac{\kappa}{2}\sqrt{3} \\ -j\frac{\mu}{2}-\frac{\kappa}{2}\sqrt{3} & -j\frac{\mu}{2}+\frac{\kappa}{2}\sqrt{3} & j\mu \end{bmatrix} \quad (3)$$

where

- ω angular frequency;
- G geometry factor [7];
- μ and κ elements of the well-known Polder tensor [10].

If ferrite losses are not taken into account, i.e., Im (μ) and Im (κ) = 0, the matrix of (3) satisfies the condition in (2), i.e., it represents a lossless three-port. In order for this three-port to act as a circulator all three ports have to be matched. Since the structure is symmetrical, this condition can only be satisfied if all three ports are terminated by the same impedance. It follows then that if two of the ports are terminated in matching impedances the input impedance at the third port is the complex conjugate of the terminating impedances. Assuming infinite isolation at port 3, the input impedance of the circuit in Fig. 1(b) can be computed by setting

$$V_3 = i_3 = 0. \quad (4)$$

In our specific case it is most convenient to use admittances instead of impedances. Subject to the condition of (4), this input admittance, in terms of (1), becomes [8]

$$Y_{\text{in}} = (\alpha - \beta^2/\gamma)^{-1}. \quad (5)$$

If the sense of circulation assumed in (4) corresponds to the actual sense of circulation for the junction, the calculated real part of Y_{in} will be positive. If the wrong sense of circulation was assumed, the calculated real part of Y_{in} will be negative.

Magnetic losses can be taken into account by using the complex frequency and field-dependent elements of the Polder tensor, i.e.,

$$\mu = \mu' - j\mu''$$

$$\kappa = \kappa' - j\kappa''.$$

The functional dependence of μ and κ on frequency and field for different values of $4\pi M$ can be found in the literature [11]. For the lossy junction, where maximum isolation and maximum return loss do not necessarily coincide [12], condition (5) implies that we define the circulator at the point of infinite isolation. This will be further discussed in Section II.

Applying (3)–(5) to the lossy junction in Fig. 1 results in

$$Y_{\text{in}} = \left(\frac{\sqrt{3}}{G\omega\mu_{\text{eff}}}\frac{\kappa}{\mu} - \frac{j}{\omega G\mu_{\text{eff}}}\right) \quad (6)$$

where

$$\mu_{\text{eff}} = \frac{\mu^2 - \kappa^2}{\mu}.$$

The input admittance from (6), with arbitrarily fixed values for the geometry factor G and saturation magnetization $4\pi Ms$, is shown in Figs. 2 and 3. In Fig. 2 the magnetic biasing field is held constant and frequency is the variable. In Fig. 3 frequency is a constant and biasing field is the variable. This type of curve can be used in the design of circulators in the following way. One first finds the frequency or field at which the real part of the input admittance equals the characteristic admittance of the connecting transmission line. The imaginary part of the admittance at this frequency or field is the conjugate of the susceptance required for a match to the transmission line and hence for circulation. Additional curves of this type showing the effects of varying the ferrite disk thickness are included in [1]. It should be kept in mind that, for the model, a certain sense of circulation was assumed, and a change of sign of Re (Y_{in}) corresponds to a change in direction of circulation; i.e., a negative sign indicates that the original choice of direction of circulation was wrong. In Figs. 2 and 3 the inversion of the sign of Re (Y_{in}) is consistent with the well-known fact that conventional above- and below-resonance circulators circulate in opposite directions.

Using the circulator condition of (4), the formulation used in (5) is general enough to be applied to any lumped-element circulator structure if its impedance matrix is known.[1] In particular, it can be applied to the

[1] Reference [3] derives the impedance matrices for different basic lumped-element circulator configurations.

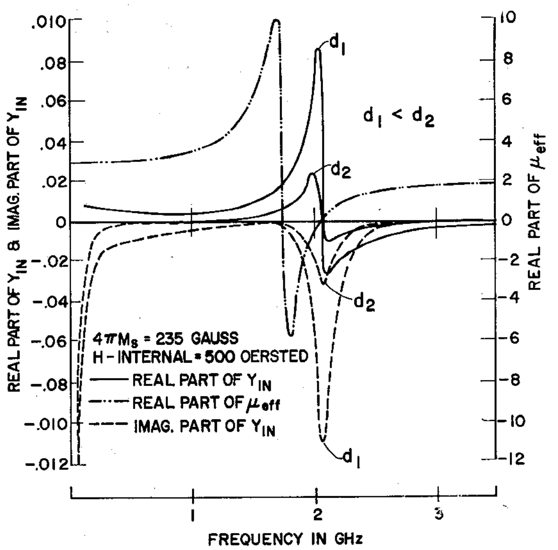


Fig. 2. Input admittance as a function of the applied frequency for different substrate thicknesses d_n.

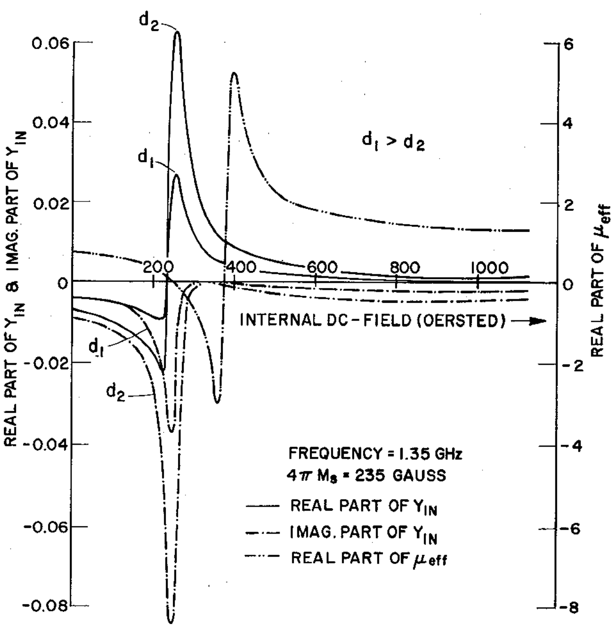


Fig. 3. Input admittance Y_{in} of the lumped-element circulator as a function of the internal dc field for different substrate thicknesses d_n.

structure in Fig. 4 which involves the addition of a capacitor C_0 and which will be shown to be capacitively switchable. The impedance matrix of the structure in Fig. 4 is given by [3]

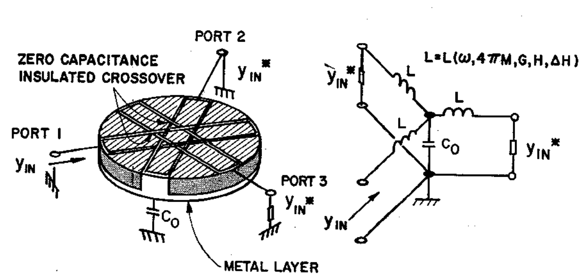


Fig. 4. Lumped-element circulator junction and approximate equivalent circuit with common capacitor C_0.

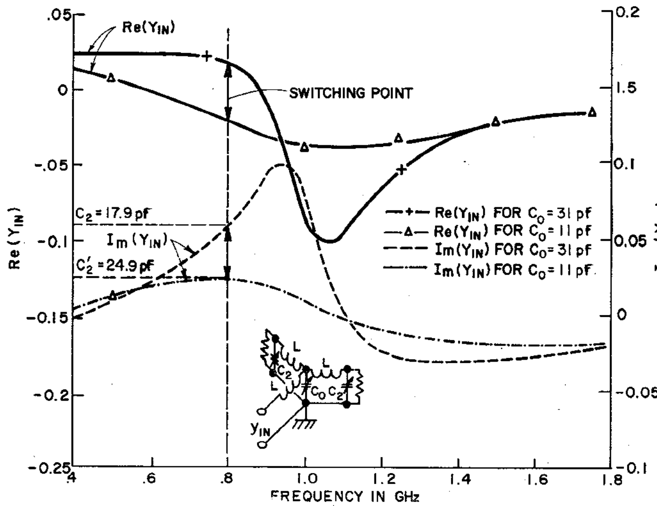


Fig. 5. Input admittance of lumped-element circulator for different values of C_0 ($4\pi M = 235$ G; $H_{int} = 150$ Oe; $\Delta H = 35$ Oe).

where $s = j\omega$.

Z can, therefore, be rewritten as

$$[Z] = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \gamma_1 & \alpha_1 & \beta_1 \\ \beta_1 & \gamma_1 & \alpha_1 \end{bmatrix} \tag{8}$$

and under the condition of (4)

$$Y_{in} = \left(\alpha_1 - \frac{\beta_1^2}{\gamma_1}\right)^{-1}. \tag{9}$$

This input admittance is plotted in Fig. 5 as a function of frequency for different values of the capacitance C_0. For this geometry a change in the value of the capacitance C_0 from approximately 31 to 11 pF causes a change in Re (Y_{in}) at 0.8 GHz from approximately $+1/50\ (\Omega)^{-1}$ to $-1/50\ (\Omega)^{-1}$.

$$[Z] = \begin{bmatrix} j\omega G\mu + \frac{1}{j\omega C_0} & \omega G\left(-\frac{\mu}{2} - \frac{\kappa}{2}\sqrt{3}\right) + \frac{1}{j\omega C_0} & \omega G\left(-j\frac{\mu}{2} + \frac{\kappa}{2}\sqrt{3}\right) + \frac{1}{j\omega C_0} \\ \omega G\left(-j\frac{\mu}{2} + \frac{\kappa}{2}\sqrt{3}\right) + \frac{1}{j\omega C_0} & j\omega G\mu + \frac{1}{j\omega C_0} & \omega G\left(-j\frac{\mu}{2} - \frac{\kappa}{2}\sqrt{3}\right) + \frac{1}{j\omega C_0} \\ \omega G\left(-j\frac{\mu}{2} - \frac{\kappa}{2}\sqrt{3}\right) + \frac{1}{j\omega C_0} & \omega G\left(-j\frac{\mu}{2} + \frac{\kappa}{2}\sqrt{3}\right) + \frac{1}{j\omega C_0} & j\omega G\mu + \frac{1}{j\omega C_0} \end{bmatrix} \tag{7}$$

In accordance with the previous discussion, the addition of the appropriate susceptance will result in a match to 50 Ω in each case, but the sense of circulation is reversed. From the Im (Y_{in}), one sees that the susceptance which must be added for a match is capacitive in both cases but of different magnitude. The required capacitances C_2 for $C_0=31$ and 11 pF are 17.9 and 24.9 pF, respectively. These considerations indicate the possibility of building a switching circulator where the switching is achieved by simultaneously changing the values of the capacitors C_0 and C_2 without changing the applied magnetic biasing field. The change in capacitance could in principle be accomplished by semiconductor devices that are low-current devices and can be switched much faster than conventional switching circulators which require high-current switching and have, therefore, an inherently lower switching speed. The change of direction of circulation has been initially observed by Thibault using variable trimmer capacitors.

In the next section, the eigenvalue analysis will be used to verify this switching principle. The eigenvalue analysis provides considerably greater insight into the mechanisms of circulation than the complex input-admittance analysis. It is applied to a modified circuit configuration not readily deducible from the foregoing analysis. The results obtained in the next section will shed additional light upon the roles of the capacitors C_0 and C_2.

II. The Eigenvalue Analysis

Any three-port satisfying the conditions in (2) and being represented by

$$[V_m] = [Z][I_m], \qquad m = 1, 2, 3 \tag{10}$$

and

$$[Z] = \begin{bmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{bmatrix}$$

has associated impedance eigenvalues which were shown in an earlier paper by the author [3] to be

$$\lambda_1 = (\alpha + \beta + \gamma)$$

$$\lambda_2 = \left(\alpha + \beta\left(-\frac{1}{2} + \frac{j}{2}\sqrt{3}\right) + \alpha\left(-\frac{1}{2} - \frac{j}{2}\sqrt{3}\right)\right)$$

$$\lambda_3 = \left(\alpha + \beta\left(-\frac{1}{2} - \frac{j}{2}\sqrt{3}\right) + \gamma\left(-\frac{1}{2} + \frac{j}{2}\sqrt{3}\right)\right). \tag{11}$$

The scattering coefficient eigenvalues can be computed from the impedance eigenvalues of (11) by

$$\lambda_n' = \frac{1 - \dfrac{\lambda_n}{Z_0}}{1 + \dfrac{\lambda_n}{Z_0}}, \qquad n = 1, 2, 3 \tag{12}$$

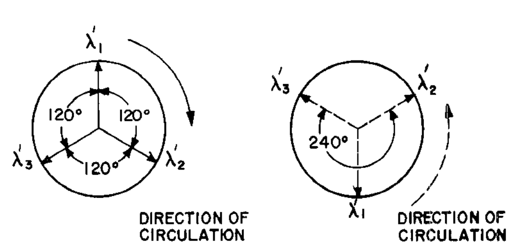


Fig. 6. Eigenvalue phases for opposite directions of circulation.

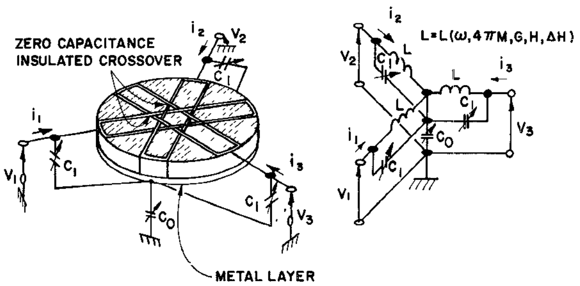


Fig. 7. Switching circulator (left) and approximate equivalent circuit (right).

where Z_0 is the characteristic impedance of the transmission lines connected to the three-port. The eigenvalues of the scattering matrix were used in the now classical papers [13], [14] on stripline and waveguide circulators. In terms of these parameters, it turns out that in the lossless case, the circulator condition described in Section I simply translates into a phase requirement on the scattering parameter eigenvalues. For circulation

$$|\angle\lambda_1' - \angle\lambda_2'| = |\angle\lambda_2' - \angle\lambda_3'| = |\angle\lambda_3' - \angle\lambda_1'| = 120° \tag{13}$$

is required. ($\angle\lambda_i'$ is the phase of λ_1'.) The direction of circulation is determined by the subscript sequence (Fig. 6). Therefore, an interchange of $\angle\lambda_1'$ and $\angle\lambda_2'$ or $\angle\lambda_3'$ and $\angle\lambda_2'$ corresponds to a change in direction of circulation.

In order to satisfy the phase requirement for circulation, one must have some means of adjusting the $\angle\lambda_i'$ relative to one another. It has been established in [2] and [3] that, for the structure of Fig. 7, the capacitance C_0 influences only $\angle\lambda_1'$, while C_1 influences only $\angle\lambda_2'$ and $\angle\lambda_3'$, and that these parameters, together with biasing field, geometry and the saturation magnetization of the ferrite, permit satisfaction of the circulator requirement over a broad band. References [2] and [6] also indicated the possibility of switching the direction of circulation by switching the values of capacitors C_0 and C_1. Figs. 8 and 9 illustrate this possibility.

Fig. 8 shows the $\angle\lambda_i'$ for a set of parameters chosen such that the circulator requirement is met at 1.25 GHz with $|\angle\lambda_2' - \angle\lambda_3'| = 120°$. In Fig. 9 no parameters except C_0 and C_1 are changed from Fig. 8; C_1 is adjusted such that $|\angle\lambda_2' - \angle\lambda_3'|$ at 1.25 GHz is increased from 120° to 240°, and C_0 is adjusted to place λ_1' midway

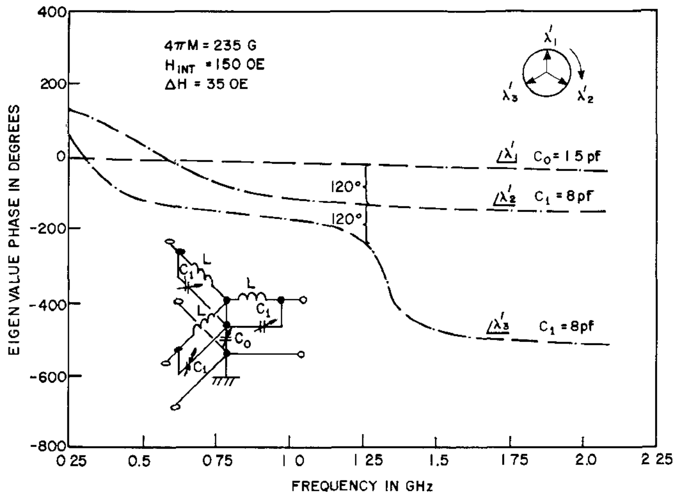
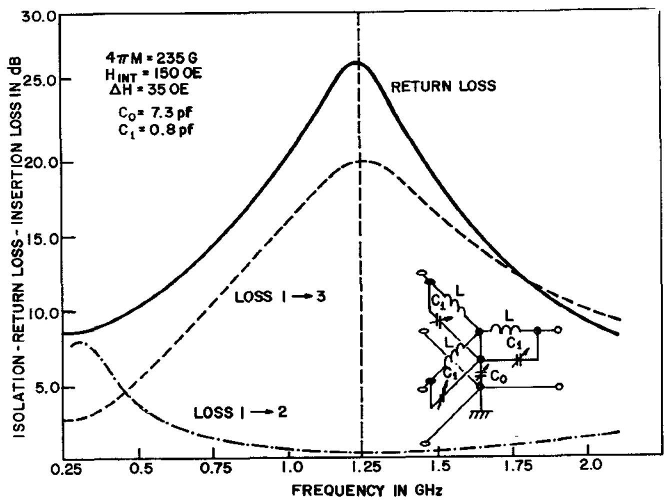


Fig. 8. Eigenvalue phases as a function of frequency for one direction of circulation.



(a)

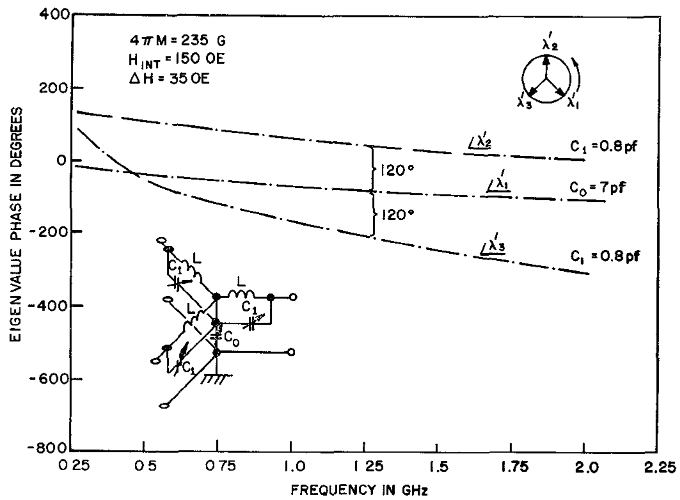
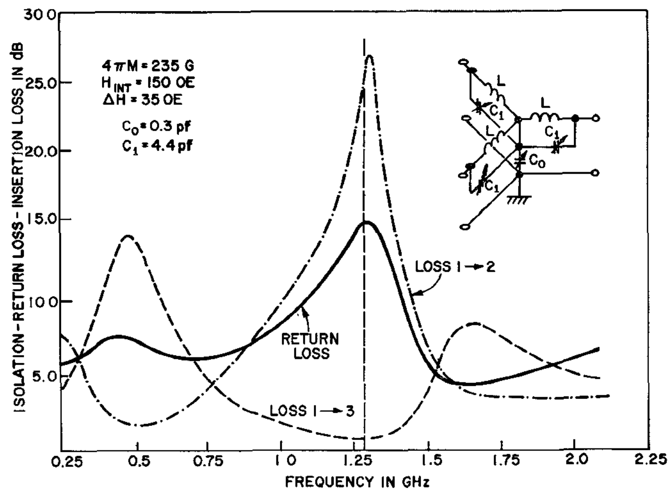


Fig. 9. Eigenvalue phases as a function of frequency for opposite direction of circulation.



(b)

Fig. 10. (a) Computed performance for circulator. (b) Computed performance for circulator with opposite direction of circulation.

between λ_2' and λ_3', thus satisfying the circulator requirement. Reading from the top down, the subscript sequences in Figs. 8 and 9 are, respectively, 1, 2, 3 and 2, 1, 3, indicating a reversal in the sense of circulation.

The circulation point in Figs. 8 and 9 assumes a lossless structure. However, when losses are present the circulator condition may no longer be determined by S-parameter eigenvalue phases alone. This is readily apparent if one considers that the requirement for a match is that the S-parameter eigenvalues sum to zero. This obviously requires the angles between eigenvalues to be other than 120° if their magnitudes are unequal. It is well to point out here that there is no one-to-one correlation between return loss and isolation, so best circulation must be defined as the best compromise between these characteristics.

The return loss and the transmission losses in the presence of magnetic loss can be computed using the following equations:

$$\text{return loss} \qquad = 20 \log |S_{11}|$$

$$\text{transmission loss} \quad \text{port } 1 \rightarrow 2 = 20 \log |S_{21}|$$

$$\text{port } 1 \rightarrow 3 = 20 \log |S_{31}|$$

where the S_{ij} are computed from the λ_n' as indicated in [3]. Fig. 10(a) is a plot of such characteristics with C_0 and C_1 adjusted for circulator action, all other parameters arbitrarily remaining unchanged from their values above. Fig. 10(b) shows calculated results for changes in C_0 and C_1 only, such that the sense of circulation is reversed. The computed S-parameter eigenvalue phases for Fig. 10(a) and (b) are shown in Fig. 11(a) and (b), respectively. It is obvious that the phase separations at the operating frequency 1.25 GHz are not 120°. This departure from 120°, together with the difference in performance, is due to magnetic loss that affects the magnitudes of the eigenvalues as shown in Fig. 12. For $C_1 = 4.4$ pF, corresponding to Figs. 10(b) and 11(b), the losses in the region of operation of the circulator are

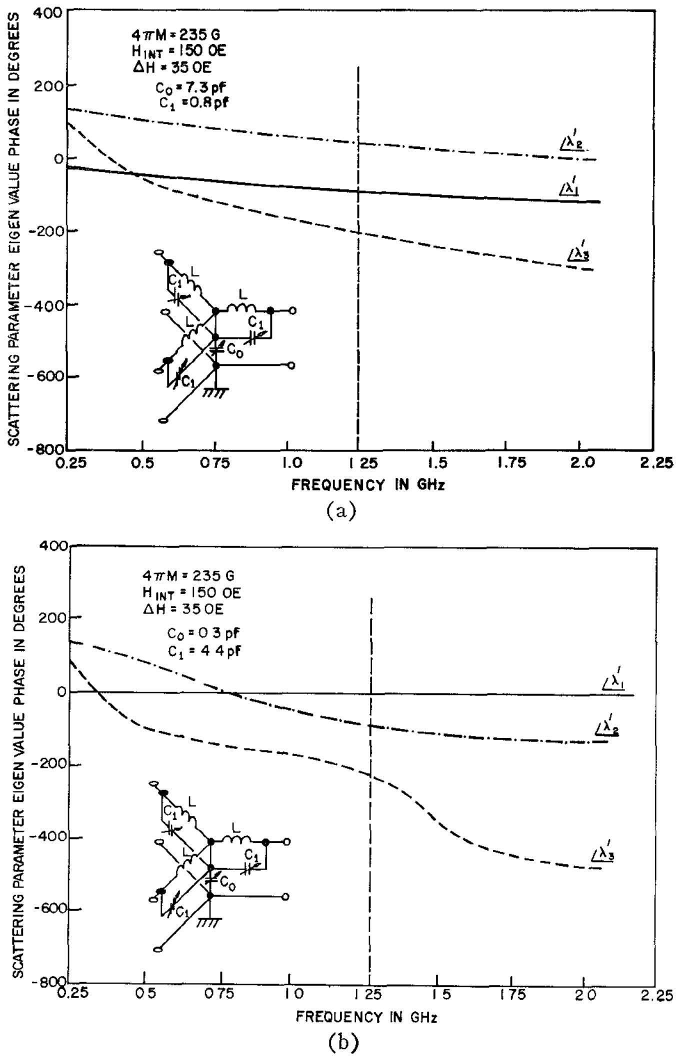


Fig. 11. (a) Eigenvalue phase of circulator in Fig. 10(a). (b) Eigenvalue phase of circulator in Fig. 10(b).

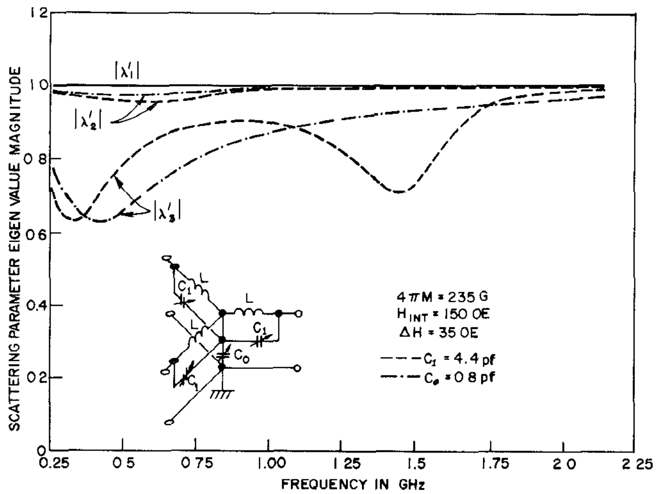


Fig. 12. Eigenvalue magnitude of circulators in Fig. 10(a) and (b).

high as compared to those for $C_1 = 0.8$ pF, corresponding to Figs. 10(a) and 11(a). There are several possible ways of minimizing this loss and improving performance [3], but they will not be pursued at this time since, within the scope of this paper, only a demonstration of the switching principle is intended.

At this point it seems useful to compare the structure in Figs. 4 and 5 with the one used in Fig. 7. C_2 of Fig. 5 influences all three eigenvalue phases, while C_1 of Fig. 7 as pointed out before influences only $\angle\lambda_2'$ and $\angle\lambda_3'$, and therefore permits an independent adjustment of the eigenvalues which is not possible in the structure of Fig. 5. This added degree of freedom facilitates the realization of the switching condition and permits improved circulator characteristics.

Conclusion

Two independent analytical methods applied to the lumped-element circulator show the possibility of switching circulators by changing two capacitance values (four capacitors) while keeping the magnetic biasing field constant. It should be pointed out that, in principle, any type of circulator can be made to reverse its sense of circulation by a change of parameters other than the magnetic biasing field. For other than the lumped-element circulators, however, this seems to be a remote possibility at this time.

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